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Optimal solution of Fuzzy Transportation Problem Using Hexagonal Fuzzy Numbers

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Abstract— In this paper we introduce a fuzzy transportation problem with hexagonal fuzzy numbers. In which the transportation parameters like demand, supply and transportation cost are hexagonal fuzzy numbers. Vogel's approximation method is used to obtain a fuzzy basic feasible solution and the optimal solution is obtained by fuzzy zero point method. These procedures are illustrated with numerical example. The minimum hexagonal fuzzy total transportation cost with maximum membership value is shown graphically and compared.

Index Terms— Fuzzy Number, Hexagonal fuzzy number, Fuzzy Transportation problem, Initial Basic Feasible Solution, Optimal Solution

1 Introduction

THE transportation problem is the one of the subclasses of LPPs in which the objective is to transport various quantities of a single homogeneous commodity that are spread at various sources to different distances in such a way that the total transportation cost is minimum. Many algorithms have been developed to solve the transportation problem in which the cost coefficients, the demand and supply quantities are known exactly. But in the real life the parameters of the transportation problem are not always exactly known and stable. This uncertainty leads to fuzzy transportation problem. The quantities are uncertain due to many uncontrollable factors like

- Climate and weather conditions like snow, flood, rain etc
- Road hazards and traffic
- Penalty or extra cost due to delivery time or safety of delivery

To deal with imprecision in decision making Bellmann and Zadeh [3,16] introduced the concept of fuzziness. The transportation problem was originally developed by Hitchcock. A Fuzzy Transportation Problem [FTP] is a problem in which the transportation cost, demand and supply quantities are fuzzy quantities. The aim of a FTP is to find the transporting plan to minimize the total fuzzy transportation cost while satisfying fuzzy demand and supply limits. In 1982, O'heigeartaigh [11] proposed an algorithm to solve FTP with triangular membership function. In 1996, Chanas and Kutcha [4] proposed a method to find the optimal solution to the transportation problem with fuzzy coefficients. In 2003, Saad and Abbas [12] discussed an algorithm to solve a transportation problem in fuzzy environment. In 2006, Gani and Razak [7] discussed a two stage cost minimizing fuzzy transportation in which the demand and supply quantities are trapezoidal fuzzy numbers. In 2009, Dinagar and Palanivel [5] studied FTP with trapezoidal fuzzy numbers and they developed a method to find optimal solution in terms of fuzzy numbers. In 2010, Pandian and Natarajan [12] proposed a new algorithm namely fuzzy zero point method to find optimal solution of a FTP with trapezoidal fuzzy numbers. Zimmermann [17,18] showed that solutions obtained by fuzzy linear programming are always efficient.

In this paper, we introduce a FTP in which all the parameters are hexagonal fuzzy numbers. The IBFS and optimal solutions were obtained by suitable algorithms. The solution procedure is illustrated with suitable example. The hexagonal fuzzy solutions with their membership values are shown graphically.

2 Preliminaries

2.1 Definition (Fuzzy set [FS])

Let X be a nonempty set. A fuzzy set $\bar{A}$ of X is defined as $\bar{A} = \{(x, \mu_{\bar{A}}(x)) / x \in X\}$ where $\mu_{\bar{A}}(x)$ is called the membership function which maps each element of X to a value between 0 and 1.

2.2 Definition (Fuzzy Number [FN])

A fuzzy number is a generalization of a regular real number and which does not refer to a single value but rather to a connected a set of possible values, where each possible value has its weight between 0 and 1. This weight is called the membership function.

A fuzzy number $\tilde{A}$ is a convex normalized fuzzy set on the real line R such that:

- There exist atleast one $x \in \mathrm{R}$ with $\mu_{\tilde{A}}(x) = 1$
- $\mu_{\tilde{A}}(x)$ is piecewise continuous

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2.3 Definition (Triangular Fuzzy Numbers [TFN])

A fuzzy number $\tilde{A}$ is a TFN [8] denoted by (a_1, a_2, a_3) where a_1, a_2 and a_3 are real numbers and its membership function is given below,

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & \text{for } a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, & \text{for } a_2 \le x \le a_3 \\ 0, & \text{otherwise} \end{cases}$$

2.4 Definition (Trapezoidal Fuzzy Numbers [TrFN])

A fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$ is a TrFN [1] where a_1, a_2, a_3 and a_4 are real numbers and its membership function is given below,

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & \text{for } a_1 \le x \le a_2 \\ 1, & \text{for } a_2 \le x \le a_3 \\ \dfrac{a_4 - x}{a_4 - a_3}, & \text{for } a_3 \le x \le a_4 \\ 0, & \text{otherwise} \end{cases}$$

3 HEXAGONAL FUZZY NUMBERS

3.1 Definition (Hexagonal fuzzy number [HFN])

A fuzzy number $\tilde{A}_H$ is a HFN [14] denoted by $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ where a_1, a_2, a_3, a_4, a_5 and a_6 are real numbers and its membership function is given below,

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{1}{2}\left(\dfrac{x - a_1}{a_2 - a_1}\right), & \text{for } a_1 \le x \le a_2 \\ \dfrac{1}{2} + \dfrac{1}{2}\left(\dfrac{x - a_2}{a_3 - a_2}\right), & \text{for } a_2 \le x \le a_3 \\ 1, & \text{for } a_3 \le x \le a_4 \\ 1 - \dfrac{1}{2}\left(\dfrac{x - a_4}{a_5 - a_4}\right), & \text{for } a_4 \le x \le a_5 \\ \dfrac{1}{2}\left(\dfrac{a_6 - x}{a_6 - a_5}\right), & \text{for } a_5 \le x \le a_6 \\ 0, & \text{otherwise} \end{cases}$$

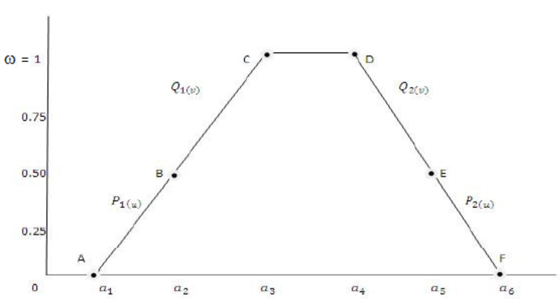

Figure 1. Graphical representation of a HFN

3.2 Definition

An HFN [9] denoted by $\tilde{A}_\omega$ is defined as $\tilde{A}_\omega = (P_1(u), Q_1(v), Q_2(v), P_2(u))$ for $u \in [0, 0.5]$ and $v \in [0.5, \omega]$.

From Fig.1

(i) $P_1(u)$ is a bounded left continuous non decreasing function over [0,0.5]

(ii) $Q_1(v)$ is a bounded left continuous non decreasing function over [0.5, ω]

(iii) $Q_2(v)$ is a bounded continuous non increasing function over [ω, 0.5]

(iv) $P_2(u)$ is increasing function a bounded left continuous non over [0.5,0]

3.2.1 Remark

In $\tilde{A}_\omega$, ω represents the maximum membership value. If ω=1 then the HFN is called a normal HFN.

3.2.2 Remark

A HFN is denoted as $\tilde{A}_\omega = (P_1(u), Q_1(v), Q_2(v), P_2(u))$ for $u \in [0, 0.5]$ and $v \in [0.5, \omega]$ where

$$P_1(u) = \frac{1}{2}\left[\frac{u - a_1}{a_2 - a_1}\right]$$

$$Q_1(v) = \frac{1}{2} + \frac{1}{2}\left[\frac{v - a_2}{a_3 - a_2}\right]$$

$$Q_2(v) = 1 - \frac{1}{2}\left[\frac{u - a_4}{a_5 - a_4}\right]$$

$$P_2(u) = \frac{1}{2}\left[\frac{a_6 - u}{a_6 - a_5}\right]$$

3.3 Definition (Positive and Negative HFN)

A HFN $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ is positive [10] if $a_i > 0 \; for \; i = 1, 2, \ldots, 6$ and it is negative if $a_i < 0 \; for \; i = 1, 2, \ldots, 6$.

3.4 Definition (Arithmetic operations on HFN)

If $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6)$, $\bar{B}_H = (b_1, b_2, b_3, b_4, b_5, b_6)$ are two HFN's [1,2,6] then the following three operations can be performed as follows:

- Addition:

$$\tilde{A}_H + \tilde{B}_H = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6)$$

- Subtraction:

$$\tilde{A}_H - \tilde{B}_H = (a_1 - b_6, a_2 - b_5, a_3 - b_4, a_4 - b_3, a_5 - b_2, a_6 - b_1)$$

- Multiplication:

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$$\tilde{A}_H * \tilde{B}_H = (a_1 * b_1, a_2 * b_2, a_3 * b_3, a_4 * b_4, a_5 * b_5, a_6 * b_6)$$

3.5 Definition (Magnitude of a HFN)

The magnitude of a HFN [15] $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ is defined as $Mag(\tilde{A}_H) = \dfrac{(2a_1 + 3a_2 + 4a_3 + 4a_4 + 3a_5 + 2a_6)}{18}$

4 FUZZY TRANSPORTATION PROBLEM [FTP]

Consider a FTP with m sources and n destinations with HFN's. The mathematical formulation of the FTP whose parameters are HFN's under the case that the total supply is equivalent to the total demand is given by:

$$Minimize\ \tilde{Z} = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} \tilde{x}_{ij} \text{ Subject to}$$

$$\sum_{j=1}^{n} \tilde{x}_{ij} \approx \tilde{a}_i \quad i = 1, 2, ..., m$$

$$\sum_{i=1}^{m} \tilde{x}_{ij} \approx \tilde{b}_j \quad j = 1, 2, ..., n$$

$$\sum_{i=1}^{m} \tilde{a}_i \approx \sum_{j=1}^{n} \tilde{b}_j \quad i = 1, 2, ..., m \text{ and } j = 1, 2, ..., n$$

and $\tilde{x}_{ij} \geq \tilde{0}$.

In which the transportation costs $\tilde{c}_{ij}$, supply $\tilde{a}_i$ and demand $\tilde{b}_j$ are hexagonal fuzzy quantities.

4.1. Algorithm (Vogel's Approximation Method)

The Vogel's Approximation Method (VAM) is an iterative method for finding an initial fuzzy basic feasible solution for FTP. The method proceeds as follows.

Step 1: Calculate the magnitude of difference between the minimum and next to minimum transportation cost in each row and column and write it as "Diff." along the side of the table against the corresponding row/column.

Step 2: In the row /column corresponding to maximum "Diff.", make the maximum allotment at the box having minimum transportation cost in that row/ column.

Step 3: If the maximum "diff." corresponding to two or mo rows or columns are equal, select the top most row and the extreme left column.

4.2. Algorithm (Fuzzy Zero Point Method)

The Fuzzy zero point Method [12] is used for finding fuzzy optimal solution for FTP and it proceeds as follows.

Step 1: Construct the fuzzy transportation table for the given FTP and then, convert it into a balanced one, if it is not.

Step 2: Subtract each row entries of the fuzzy transportation table from the row minimum.

Step 3: Subtract each column entries of the resulting fuzzy transportation table after using the Step 2. from the column minimum.

Step 4: Check if each column fuzzy demand is less to the sum of the fuzzy supplies whose reduced costs in that column are fuzzy zero. Also, if each row fuzzy supply is less to the sum of the column fuzzy demands whose reduced costs in that row are fuzzy zero. If so, go to Step 7. Otherwise go to Step 5.

Step 5: Draw the minimum number of horizontal and vertical lines to cover all the fuzzy zeros of reduced fuzzy transportation table.

Step 6: Construct the new revised fuzzy transportation table as follows:

(i) Find the smallest entry of the reduced fuzzy cost matrix not covered by any lines.

(ii) Subtract this entry from all the uncovered entries and add the same to all entries at the intersection of any two lines.

And then go to Step 4.

Step 7: Select a cell in the reduced fuzzy transportation table whose reduced cost is the maximum cost. Say (x, y). If there is more than one, then select anyone.

Step 8: Select a cell in the x-row or/and y-column of the reduced fuzzy transportation table which is the only cell whose reduced cost is fuzzy zero and then, allot the maximum possible to that cell. If such cell does not occur for the maximum value, find the next maximum so that such a cell occurs. If such cell does not occur for any value, we select any cell in the reduced fuzzy transportation table whose reduced cost is fuzzy zero.

Step 9: Revise the reduced fuzzy transportation table after deleting the fully used fuzzy supply points and theDestin fully received fuzzy demand points and also, modify it to include the not fully used fuzzy supply points and the not fully received fuzzy demand points.

Step 10: Repeat the steps 7 to 9 until all fuzzy supply points are fully used and fuzzy demand points are fully received.

Step 11: This allotment gives a fuzzy solution to the given fuzzy transportation problem.

4.3 Numerical Example

Consider the following FTP with Hexagonal fuzzy numbers.

		Destinations				Supply
		D1	D2	D3	D4	
Origins	O1	(3,7,11, 15,19,24)	(13,18,23, 28,33,40)	(6,13,20, 28,36,45)	(15,20,25, 31,38,45)	(7,9, 11,13, 16,20)
	O2	(16,19,24, 29,34,39)	(3,5,7, 9,10,12)	(5,7,10, 13,17,21)	(20,23,26, 30,35,40)	(6,8, 11,14, 19,25)
	O3	(11,14,17, 21,25,30)	(7,9,11, 14,18,22)	(2,3,4, 6,7,9)	(5,7,8, 11,14,17)	(9,11, 13,15, 18,20)
Demand		(3,4,5, 6,8,10)	(3,5,7, 9,12,15)	(6,7,9, 11,13,16)	(10,12,14, 16,20,24)	

Solution:

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The fuzzy IBFS of the above FTP can be obtained by VAM as follows:

Now using the Step 1 of the VAM calculate the difference for each row and column.

TABLE 1

		Destinations				Supply	Diff.
		D1	D2	D3	D4		
Origins	O1	(3,7,11, 15,19,24)	(13,18,23, 28,33,40)	(6,13,20, 28,36,45)	(15,20,25, 31,38,45)	(7,9, 11,13, 16,20)	11.38
	O2	(16,19,24, 29,34,39)	(3,5,7, 9,10,12)	(5,7,10, 13,17,21)	(20,23,26, 30,35,40)	(6,8, 11,14, 19,25)	4.27
	O3	(11,14,17, 21,25,30)	(7,9,11, 14,18,22)	(2,3,4, 6,7,9)	(5,7,8, 11,14,17)	(9,11, 13,15, 18,20)	5.06
Demand		(3,4,5, 6,8,10)	(3,5,7, 9,12,15)	(6,7,9, 11,13,16)	(10,12,14 16,20,24)		
Diff.		6.38	5.56	6.89	18.61 ▲		

Using the step 2 identify the highest difference. In this case it occurs at column 4. Now allocate the maximum possible units (9,11,13,15,18,20) to the minimum cost position (3, 4) and write the remaining in column 4. After removing the third row and then by repeating the steps 1 and 2, the highest difference occurs at second column. Now allocate the maximum possible units (3,5,7,9,12,15) to the minimum cost position (2,2) and write the remaining in row 2. After removing the second column repeats the step 1, we obtain the table given below.

TABLE 2

		Destinations				Supply	Diff.
		D1	D2	D3	D4		
Origins	O1	(3,7,11, 15,19,24)	-	(6,13,20, 28,36,45)	(15,20,25, 31,38,45)	(7,9, 11,13, 16,20)	11.38
	O2	(16,19,24, 29,34,39)	**(3,5,7, 9,12,15)**	(5,7,10, 13,17,21)	(20,23,26, 30,35,40)	(-9,- 4,2,7,14 ,22)	4.27
	O3	-	-	-	**(9,11,13, 15,18,20)**	-	
Demand		(3,4,5, 6,8,10)		(6,7,9, 11,13,16)	(-10,-6, -1,3,9,15)		
Diff.		13.6 ▲	-	12.5	0		

In the above Table 2 the highest difference occurs at first column. Now allocate the maximum possible units (3,4,5,6,8,10) to the minimum cost position (1,1) and write the remaining in row 1. After removing the first column and then by repeating the step 1 and step 2, the highest difference occurs at second row. Now allocate the maximum possible units (-9,-4,2,7,14,22) to the minimum cost position (2,3) and write the remaining in column 3. After removing the second row repeats the step 1, we obtain the following table.

TABLE 3

		Destinations				Supply
		D1	D2	D3	D4	
Origins	O1	**(3,4,5, 6,8,10)**	-	(6,13,20, 28,36,45)	(15,20,25, 31,38,45)	(-3,1, 5,8, 12,17)
	O2	-	**(3,5,7, 9,12,15)**	**(-9,-4,2, 7,14,22)**		
	O3	-	-	-	**(9,11,13, 15,18,20)**	
Demand		-	-	(-16,-7,2, 9,17,25)	(-10,-6,-1, 3,9,15)	

Now allocate the remaining, we get following complete allocation table.

TABLE 4

		Destinations				Supply
		D1	D2	D3	D4	
Origins	O1	**(3,4,5, 6,8,10)**	-	**(-16,-7,2, 9,17,25)**	**(-3,1,5, 8,12,17)**	
	O2		**(3,5,7, 9,12,15)**	**(-9,-4,2, 7,14,22)**	-	
	O3	-	-	-	**(9,11,13, 15,18,20)**	
Demand		-	-	-	-	-

Therefore, the fuzzy IBFS in terms of HFNs for the given FTP is given by,

$$\tilde{x}_{11} \approx (3,4,5,6,8,10)\ ,\ \tilde{x}_{13} \approx (-16,-7,2,9,17,25)\ ,$$
$$\tilde{x}_{14} \approx (-3,1,5,8,12,17)\ ,\ \tilde{x}_{22} \approx (3,5,7,9,12,15)\ ,$$
$$\tilde{x}_{23} \approx (-9,-4,2,7,14,22)\ ,\ \tilde{x}_{34} \approx (9,11,13,15,18,20)$$

And the total fuzzy transportation minimum cost is given by

Minimize $\tilde{Z}$ = (3,4,5,6,8,10) (3,7,11,15,19,24)+
(-16,-7,2,9,17,25) (6,13,20,28,36,45)+
(-3, 1, 5, 8, 12, 17) (15,20,25,31,38,45) +
(3,5,7,9,12,15) (3,5,7,9,10,12)+
(-9,-4,2,7,14,22)(5,7,10,13,17,21)+
(9,11,13,15,18,20) (5,7,8,11,14,17)
= (-123, 31, 393, 927, 1830, 3112)

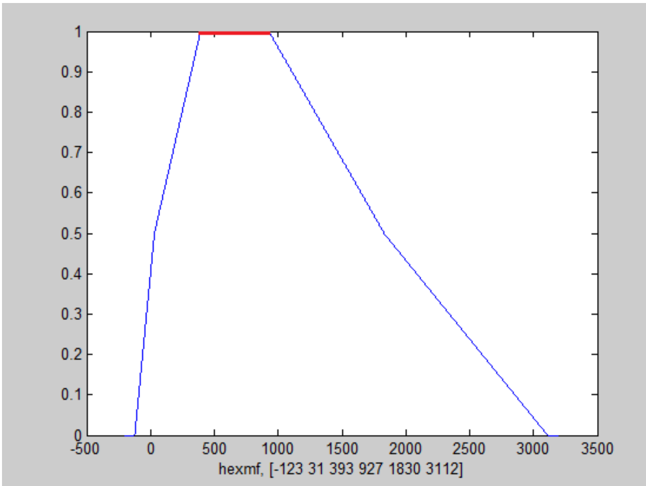

Figure 2. Graphical representation of IBFS

4.3.1 Discussions

In the above IBFS, the total fuzzy cost is (-123,31,393,927,1830,3112). And the crisp value of the IBFS is 935.6. From the figure 2 , the total cost various from 375 to 950 occurs with maximum membership value.

4.4 Numerical Example

The optimum solution by fuzzy zero point method is illustrated by the following example.

		Destinations				Supply
		D1	D2	D3	D4	
Origins	O1	(3,7,11, 15,19,24)	(13,18,23, 28,33,40)	(6,13,20, 28,36,45)	(15,20,25, 31,38,45)	(7,9, 11,13, 16,20)
	O2	(16,19,24, 29,34,39)	(3,5,7, 9,10,12)	(5,7,10, 13,17,21)	(20,23,26, 30,35,40)	(6,8, 11,14, 19,25)
	O3	(11,14,17, 21,25,30)	(7,9,11, 14,18,22)	(2,3,4, 6,7,9)	(5,7,8, 11,14,17)	(9,11, 13,15, 18,20)
De-mand		(3,4,5, 6,8,10)	(3,5,7, 9,12,15)	(6,7,9, 11,13,16)	(10,12, 14,16, 20,24)	

Solution:

Here the total fuzzy supply and the total fuzzy demand are equal. Therefore the given problem is balanced one. Now using the Step 2 and Step 3 of the fuzzy zero point method, we get the following table.

TABLE 5

		Destinations				Sup ply
		D1	D2	D3	D4	
Origins	O1	0	(-11,-1,8, 17,26,37)	(-18,-6,5, 17,29,42)	(-24,-10,3, 18,31,46)	(7,9, 11,13, 16,20)
	O2	(4,9,15, 22,29,36)	0	(-7,-3,1, 6,12,18)	(-7,2,10, 21,30,41)	(6,8, 11,14, 19,25)
	O3	(2,7,11, 17,22,28)	(-2,2,5, 10,15,20)	0	0	(9,11, 13,15, 18,20)
De-mand		(3,4,5, 6,8,10)	(3,5,7, 9,12,15)	(6,7,9, 11,13,16)	(10,12,14, 16,20,24)	

The above table does not satisfy the Step 4.Therefore using the steps 5 and 6 for the above table. And then using the Step 4 to the Step 6 of the fuzzy zero point method, we have the following allotment table.

TABLE 6

		Destinations				Sup-ply
		D1	D2	D3	D4	
Origins	O1	0	(-64,-35,-9, 20,48,79)	(-89,-52,-18, 19,54,91)	0	(7,9, 11,13, 16,20)
	O2	(-38,-13,12, 39,63,89)	0	0	(-25,-10,4, 20,33,48)	(6,8, 11,14, 19,25)
	O3	(-47,-18,9, 40,68,99)	(-9,-1,6, 16,27,38)	0	0	(9,11, 13,15, 18,20)
De-mand		(3,4,5, 6,8,10)	(3,5,7, 9,12,15)	(6,7,9, 11,13,16)	(10,12,14, 16,20,24)	

Now using the allotment rules of the fuzzy zero point method, we have the following table.

TABLE 7

		Destinations				Supply
		D1	D2	D3	D4	
Origins	O1	**(3,4,5, 6,8,10)**			**(-3,1,5, 8,12,17)**	(7,9,11, 13,16,20)
	O2		**(3,5,7, 9,12,15)**	**(-9,-4,2, 7,14,22)**		(6,8,11, 14,19,25)
	O3			**(-16,-7,2, 9,17,25)**	**(-16,-6,4, 13,25,36)**	(9,11,13, 15,18,20)
De-mand		(3,4,5, 6,8,10)	(3,5,7, 9,12,15)	(6,7,9, 11,13,16)	(10,12,14, 16,20,24)	

Therefore, the fuzzy optimal solution for the given FTP is

$$\tilde{x}_{11} \approx (3,4,5,6,8,10)\ ,\ \tilde{x}_{14} \approx (-3,1,5,8,12,17)\ ,$$
$$\tilde{x}_{22} \approx (3,5,7,9,12,15)\ ,\tilde{x}_{23} \approx (-9,-4,2,7,14,22)\ ,$$
$$\tilde{x}_{33} \approx (-16,-7,\ 2,9,17,25)\ ,\tilde{x}_{34} \approx (-16,-6,4,13,25,36)$$

Hence the total minimum cost is given by

$Minimize\ \tilde{Z}$ = (3,4,5,6,8,10) (3,7,11,15,19,24) +
(-3,1,5,8,12,17) (5,20,25,31,38,45)+
(3,5,7,9,12,15) (3,5,7,9,10,12) +
(-9,-4,2,7,14,22) (5,7,10,13,17,21)+
(-16,-7, 2,9,17,25) (2,3,4,6,7,9)+
(-16,-6,4,13,25,36) (5,7,8,11,14,17)
=(-184,-18, 289, 707, 1435, 2484)

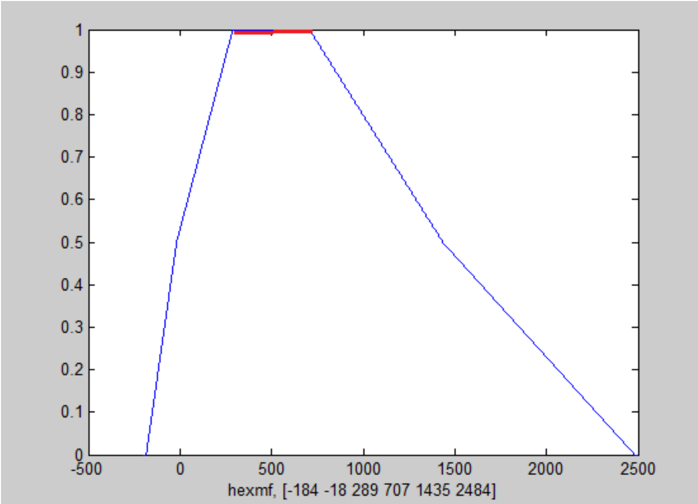

Figure 3. Graphical representation of the optimum solution

4.4.1. Discussions

The optimal solution of the given hexagonal fuzzy transportation problem is (-184,-18,289,707,1435,2484). And its crisp value is 713. From the above fige 3. the total cost varies from 260 to 740 occurs with maximum membership value.

5 Conclusion

Recently many researchers studied on the solution of FTP with triangular fuzzy numbers and trapezoidal fuzzy numbers. In the present paper we introduced FTP with hexagonal fuzzy numbers and we obtained crisp as well as fuzzy IBFS and the optimum solution for it. The arithmetic operations on hexagonal fuzzy numbers are used to find the solutions. By introducing hexagonal fuzzy numbers instead of triangular or trapezoidal fuzzy numbers, we can reduce fuzziness in the solution. Hence, this will be helpful for decision makers who are handling logistic and supply chain problems in fuzzy environment. For future research we propose effective implementation of the hexagonal fuzzy numbers in all fuzzy problems.

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